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# COST BASED ACCEPTANCE SAMPLING PLANS AND PROCESS CONTROL SCHEMES

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## ABSTRACT

A review of procedures for designing acceptance sampling plans and process control schemes according to economic criteria is given. The use of nonstatistical criteria to design these procedures has been the subject of considerable research, and a number of different process models and procedures have been developed. A comparison of the major approaches to model formulation and design, and a discussion of the practical implementation of these techniques is the primary focus of this paper.

During the last 20 years, considerable research has been devoted to the use of economic criteria to design acceptance sampling plans and process control schemes. This paper gives an overview of the major developments and approaches. Some comparison of the different approaches to model formulation and a discussion of the practical implementation of these techniques is also given.

## INTRODUCTION

Acceptance sampling and process control are the primary statistical techniques used in a quality assurance program. These techniques have found wide application in industry, particularly in manufacturing, in the following areas: incoming material inspection, surveillance of the production process, estimation of lot or process characteristics, process capability analysis, and finished product quality auditing.

Traditionally, acceptance sampling and process control schemes are designed with respect to statistical criteria. For example, one may choose the sample size and acceptance number for a single-sampling plan for attributes so that the operating characteristic curve passes through (or near) two points specifically selected to give certain probabilities of lot rejection at specified levels of lot or process quality. Similarly, one may design a control chart so that the power of the chart to detect a particular shift in process quality and the probability of false alarms are equal to specified values. While the traditional approach often produces acceptable results, it is also possible to design acceptance sampling and process control schemes with respect to economic criteria. This has considerable intuitive appeal since there has been an increasing emphasis on quality costs in recent years. Furthermore, the use of these techniques have direct economic consequences in that one is balancing the costs associated with sampling, testing and process surveillance against internal and external failure costs. Since the design of the procedure affects these costs, it is logical to consider this design from an economic viewpoint.

## ECONOMIC MODELING AND PROCESS CONTROL

The Shewhart control chart is probably the most widely used process surveillance device. To design a control chart, we must choose the sample size ( $n$ ), the control limit ( $k$ , the multiple of  $\sigma$ ), and the interval between samples ( $h$  hours). Despite the non-optimality of fixed sample size, fixed sampling interval procedures, the Shewhart control chart has gained widespread use because of its flexibility, simplicity of administration and the additional information about process performance often contained in the pattern of points plotted on the chart.

Considerable effort has been devoted to developing economic models of Shewhart control charts. These models usually assume that the process is characterized by one in-control state that represents the mean of the quality characteristic when no assignable causes are present, and  $s \geq 1$  out-of-control states. The probability model that governs the transitions between these  $s + 1$  states is called the process failure mechanism.

## Costs and Measures of Effectiveness

Three categories of costs are usually considered in the development of economic models for Shewhart control charts: the cost of sampling and testing, usually of the form  $a_1 + a_2n$ , the costs of investigating and possibly correcting action signals, and the cost of producing defective items (internal and external failure costs). These costs are then combined to form a total cost per unit time function. The general approach is to define a cycle as the length of time  $T$  during which the process begins operating in the in-control state and eventually returns to the in-control state following a process adjustment.

If  $C$  is the cost incurred during a cycle, then the expected cost per unit time is

$$E(L) = E(C)/E(T) \quad (1)$$

Since  $E(C)$  and  $E(T)$  are functions of the design parameters of the control chart, numerical optimization (direct search) techniques can be employed to find the parameter values that minimize the expected cost per unit time.

#### Basic Process Models

The fundamental process model often used in economic design of the Shewhart control chart is due to Duncan [7]. His work was the first to deal with a fully-economic approach to control chart design and to explicitly consider the optimization problem. The Duncan model assumes a single assignable cause (thus  $s=1$ ) and that transitions between the in-control and out-of-control states occur according to a Poisson process with intensity  $\lambda$  occurrences per unit time. Thus the length of time the process remains in control, given that it starts in that state, is an exponential random variable with mean  $1/\lambda$ . This is equivalent to a process failure mechanism with a uniform hazard function.

The cycle consists of four periods: (1) the in-control period, (2) the out-of-control period, (3) the time to take a sample and interpret the results and (4) the time to find the assignable cause. Assuming that the process continues to run during searches, the expected cycle length is

$$E(T) = 1/\lambda + h/(1-\beta) - \tau + gn + D \quad (2)$$

where  $h$  is the sampling interval (in hours),  $1 - \beta$  is the power of the chart for detecting a specified out-of-control state,  $n$  is the sample size,  $g$  is the time required to take and interpret a sample,  $D$  is the expected time to find an assignable cause and

$$\tau = \int_{jh}^{(j+1)h} e^{-\lambda t} \lambda(t-jh) dt / \int_{jh}^{(j+1)h} e^{-\lambda t} \lambda dt$$

$$= [1 - (1+\lambda h)e^{-\lambda h}] / \lambda(1-e^{-\lambda h}) \quad (3)$$

is the expected time of occurrence of the shift, given that it occurs between the  $j$ th and  $(j+1)$ th samples.

Let  $V_0$  and  $V_1$  be the net income per hour of operation in the in-control and out-of-control states, respectively. The costs of investigating real and false alarms are  $a_3$  and  $a_2$ , respectively. Since there are  $E(T)/h$  samples per cycle and  $ae^{-\lambda h}/(1-e^{-\lambda h})$  false alarms per cycle, on the average, the expected net income per cycle is

$$E(C) = V_0(1/\lambda) + (a_1 + a_2 n) E(T)/h$$

$$+ V_1[h/(1-\beta) - \tau + gn + D]$$

$$- a_3 - a_3'ae^{-\lambda h} / (1-e^{-\lambda h}) \quad (4)$$

Dividing (4) by (2) gives the expected net income per hour, say

$$E(I) = V_0 - E(L) \quad (5)$$

where

$$E(L) = (a_1 + a_2 n)/h$$

$$+ \frac{1}{2} a_4 [h/(1-\beta) - \tau + gn + D]$$

$$+ a_3 + a_3'ae^{-\lambda h} / (1-e^{-\lambda h}) \left\{ \right.$$

$$\left. + [1/\lambda + h/(1-\beta) - \tau + gn + D] \right\} \quad (6)$$

and  $a_4 = V_0 - V_1$  is the penalty cost (per hour) resulting from production in the out-of-control state. Maximizing  $E(I)$  in (5) is equivalent to minimizing  $E(L)$  in (6). There are three decision parameters,  $n$ ,  $k$  and  $h$  (the function  $E(L)$  depends on the width of the control limits  $k$  through  $\alpha$  and  $\beta$ ); and nine user-supplied constants that describe the production process,  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_3'$ ,  $a_4$ ,  $\lambda$ ,  $g$ ,  $D$ , and  $\delta$  the magnitude of the process shift (which is involved in determining  $\alpha$  and  $\beta$ ).

While this basic process model could be used in many situations, there are a number of variations that are potentially important. The first of these is the incorporation of more than one out-of-control state. Duncan [8] has generalized the cost model above to a situation where there are  $s$  out-of-control states (assignable causes). Knappenberger and Grandage [13] also present a model capable of treating several out-of-control states. The Duncan model appears somewhat more realistic in that it employs different search costs for different assignable causes, while the Knappenberger and Grandage model uses an average or composite cost. On the other hand, the Knappenberger and Grandage model allows continued process deterioration beyond the initial shift, while the Duncan model allows only one or two shifts to occur. Both models are considerably more sophisticated in terms of their data requirements than (6) above. Furthermore, there is considerable evidence that a single assignable cause model that matches the true multi-state system in important ways will be a satisfactory approximation. Specifically, if we let the single out-of-control state be a weighted average of the  $s$  out-of-control states with the weights chosen proportionally to the probability of occurrence of the state, excellent results are obtained. In practice, there is probably little reason to use a multiple out-of-control state model in most situations.

The assumption of a constant hazard function for the process failure mechanism (an exponential distribution for the time between shifts) is critical. If the occurrence of assignable causes can be thought of as random "shocks" disturbing the system, then the probability of a shift occurring within any small interval of time is proportional to the length of the interval, and the exponential model is probably appropriate. However, if assignable causes occur as a result of the cumulative effects of vibration or heat, improper set-up, or excessive stresses during start-up, as is the case with many production

processes, then the exponential model may be entirely inappropriate. Distributions with either increasing or decreasing hazard functions may be more reasonable. Misspecification of this aspect of the process model may have very serious economic impact. Unfortunately, it is not straightforward to incorporate other process failure mechanisms in (6). A discrete-time, single-cause model that allows free choice of the process failure mechanism has been developed by Baker [2], but as it assumes that a sample is taken at the end of each period, the optimum intersample interval cannot be explicitly determined.

The assumptions in (6) that the process continues in operation during the search for an assignable cause and that the cost of repair is not included are unrealistic for many processes. Fortunately, (6) may be easily modified to consider these possibilities. If  $D_0$  and  $D_1$  are the expected search times for false and real alarms, respectively, and if  $\Delta$  is the cost of repair, then assuming that the process is shut down during the search leads to an expected net income per unit time of

$$E(I) = V_0 - E(L) \quad (7)$$

where

$$\begin{aligned} E(L) = & \{ (a_1 + a_2 n) [1/\lambda + h/(1-\delta) - \tau] / h \\ & + a_3 + \Delta + [a_4 e^{-\lambda h} / (1 - e^{-\lambda h})] (V_0 D_0 + a_3') \\ & + V_0 D_0 + a_4 [h/(1-\delta) - \tau] \} \\ & + [1/\lambda + h/(1-\delta) - \tau + a D_0 e^{-\lambda h} / \\ & (1 - e^{-\lambda h}) + D_1] \end{aligned} \quad (8)$$

For a detailed development of this model, see Montgomery [14]. Note that given  $a_1, a_2, a_3, a_3', \Delta, a_4, \lambda, V_0, D_0, D_1$ , and the magnitude of the process shift, (8) may be easily optimized for  $n, h$ , and  $k$  using direct search methods.

#### Major Applications

Suppose that the quality characteristic of interest is represented by the parameter  $\theta$ , and that  $x$  represents the sample statistic corresponding to  $\theta$ . There are two possible values for  $\theta$ ;  $\theta = \theta_0$  corresponding to the in-control state and  $\theta = \theta_1$  which represents the out-of-control state. Then the probability of a false alarm is

$$\alpha = 1 - P(LCL \leq x \leq UCL \mid \theta = \theta_0) \quad (9)$$

and the power of the control chart is

$$1 - \beta = 1 - P(LCL \leq x \leq UCL \mid \theta = \theta_1) \quad (10)$$

where LCL and UCL are the lower and upper control limits on the Shewhart control chart. Thus by specifying the quality characteristic, the sample statistic, and the relevant sampling distribution, the cost models of the previous section could be applied

to any type of Shewhart control chart. For example, if the quality characteristic of interest is a measurement described by the process mean, then the  $\bar{x}$ -chart would be used; thus equations (9) and (10) become

$$\alpha = 2\phi(-k)$$

and

$$1 - \beta = \phi(\delta\sqrt{n}-k) + \phi(-\delta\sqrt{n}-k)$$

respectively, where  $\phi(x)$  denotes the standard normal distribution function.

The major applications of cost models such as those in the previous section have been to the  $\bar{x}$ -chart, the  $\bar{x}/R$  chart combination, and the fraction defective or  $p$ -chart. Some work has also focused on procedures for the simultaneous control of several related quality characteristics, and the treatment of non-Shewhart control charts such as the cumulative sum control chart and the use of warning limits on the  $\bar{x}$ -chart. Almost all reported studies assume that the process failure mechanism is exponential. As noted earlier, this is a critical and sometimes unwarranted assumption. For an extensive review of the literature in this area, see Montgomery [14].

Most of the papers cited in [14] contain numerical studies and usually, a sensitivity analysis. From these results, it is possible to draw certain general conclusions about control chart design:

1. The optimum sample size is largely determined by the size of the shift, with smaller shifts requiring larger samples. Small shifts may require very large samples, possibly as large as  $n=40$  or more on an  $\bar{x}$  chart.
2. Changes in  $a_1$  and  $a_2$ , the fixed and variable costs of sampling, affect all three design parameters. Increasing the fixed cost  $a_1$  increases the interval between samples and leads usually to slightly larger samples, while increasing the variable cost  $a_2$  usually results in small, infrequent samples, but narrow control limits.
3. The penalty cost for out-of-control production  $a_4$  mainly influences the interval between samples  $h$ . Larger values of  $a_4$  imply smaller optimum values of  $h$ . A similar effect is observed by increasing  $\lambda$ , the mean number of process shifts per hour.
4. The search costs  $a_3$  and  $a_3'$  mainly affect the control limits, and have a slight effect on sample size. Larger values of  $a_3$  and  $a_3'$  result in wider control limits and larger sample sizes, as increasing search costs imply that fewer false alarms are desirable.
5. The optimum economic control chart design is relatively insensitive to estimates of the cost parameters. The cost surfaces are usually flat near the optimum, and are usually steeper near the origin, so that it is better to overestimate the control chart design parameters than to underestimate them. The optimum design is most

sensitive to errors in estimating the in-control and out-of-control states (or the magnitude of the process shift).

This last finding has important practical implications, for it is often difficult to precisely estimate costs. However, the mean time between shifts and the magnitude of the process shifts are usually more easily determined from process performance data or from the engineer's knowledge of the operating environment. Note that one should exercise caution in using arbitrarily designed control charts and empirical "rules of thumb", such as  $n=5$ ,  $k=3$  and  $h=1$  for the  $\bar{x}$ -chart. In some cases, particularly those with small shifts and large penalty costs for production in the out-of-control state, significant economic penalties may result.

#### A Semi-Economic Approach

Instead of the fully-economic modeling approach outlined above, it may be desirable in some cases to use a semi-economic scheme that blends both economic and statistical criteria directly. This could be useful in situations where the analyst is unwilling or unable to estimate all of the fully-economic model's cost parameters, or where he wishes a compromise between the fully-economic and purely statistical approaches.

One such semi-economic scheme would be to find the values of  $n$ ,  $k$ , and  $h$  that minimize the expected sampling costs per cycle, but that also have desirable average run lengths for the control chart in both the in-control and out-of-control states. Consequently, we would like to choose  $n$ ,  $k$ , and  $h$  so as to

$$\text{minimize } Z = (a_1 + a_2 n) E(T)/h \quad (11)$$

$$\text{maximize } ARL_0 = 1/\alpha \quad (12)$$

$$\text{minimize } ARL_1 = 1/(1-\beta) \quad (13)$$

where  $ARL_0$  and  $ARL_1$  are the average run lengths in the in-control and out-of-control states, respectively, and  $E(T)$  is given by equation (2). Recall that  $\alpha$  and  $1-\beta$  are functions of the sample size  $n$ , the control limit factor  $k$ , and the magnitude of the shift. Note that this is a multicriterion optimization problem, and would generally be harder to solve than the optimization problems usually associated with the economic design of control charts. However, it avoids explicit estimation of the search costs  $a_2$  and  $a_3$ , and the penalty cost for production in the out-of-control state  $a_4$ , which many analysts find difficult.

Another possible formulation is to minimize the sampling cost on a per sample basis while simultaneously maximizing  $ARL_0$  and minimizing  $ARL_1$ . Thus, (11) becomes

$$\text{minimize } Z'' = a_1 + a_2 n \quad (14)$$

for  $n \geq 1$ , and (12) and (13) are unchanged. This formulation of the problem involves only  $n$  and  $k$ .

Once the optimal  $n$  and  $k$  are determined, the interval between samples could be obtained from

$$h = \left\{ \frac{a_3 + a_1 + a_2 n}{\lambda a_4 \{(1-\beta)^{-1} - 0.5\}} \right\}^{1/2} \quad (15)$$

This is an approximation for the optimum sampling interval  $h$  given  $n$  and  $k$  suggested by Duncan [7] and Chiu and Wetherill [4].

These formulations of the optimal control chart design problem have not been extensively investigated. It would be of interest to discover how they compare with the fully-economic solutions, and with the standard designs often suggested in the literature.

#### ECONOMIC ASPECTS OF ACCEPTANCE SAMPLING

There are various types of acceptance sampling schemes; single sampling, double sampling, sequential sampling, continuous sampling, and so forth. The quality characteristic being inspected may be either an attribute or a variable. For various reasons, more attention has been given to attributes sampling, and, in particular, single sampling plans for attributes have been studied extensively. We now give some of the key results pertaining to the economic design of these plans and briefly survey some of the other related work.

An attribute single sampling plan is indexed by three numbers: the lot size  $N$ , the sample size  $n$ , and the acceptance number  $c$ . If, in a random sample of size  $n$ ,  $c$  or fewer defectives are found, the lot is accepted, while if more than  $c$  defectives are found the lot is rejected. The probability of lot acceptance is

$$P(\theta) = \sum_{d=0}^c \binom{N\theta}{d} \binom{N-N\theta}{n-d} / \binom{N}{n} \quad (16)$$

where  $\theta$  is the lot fraction defective. The plot of  $P(\theta)$  versus  $\theta$  for  $0 \leq \theta \leq 1$  is called the operating characteristic curve (or OC curve) of the plan. Traditionally, we design acceptance sampling plans (i.e., choose  $n$  and  $c$ ) so that  $P(\theta)$  passes through or near two points, such as the familiar producer's and consumer's risk points  $(AQL, \alpha)$  and  $(LTPD, \beta)$ . When  $N$  is large relative to  $n$ , the hypergeometric distribution in (16) can be replaced by the binomial.

#### Prior Distributions for Attribute Sampling

Lot fraction defective  $\theta$  is a function of two sources of variability; the variability of  $p$  the process fraction defective, and the variability of  $\theta$  about  $p$ . It is usually convenient to assume that lot quality has a mixed binomial distribution; that is, each lot is produced by a production process that is in-control at the level  $p$ , but  $p$  varies from lot to lot according to a probability distribution  $f(p)$ . The distribution  $f(p)$  is often called the prior distribution for  $p$  or the process curve. It is extremely important to note that if the process quality is stable such that  $f(p) = 1$  when  $p = p_0$

and  $f(p) = 0$  when  $p \neq p_0$ , then there is no need for sampling (this result is called Mood's theorem; see Mood [15]).

While there are many possible choices for the prior distribution for  $p$ , some of the more important are the following: the continuous beta distribution,

$$f(p) = \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} p^{u-1} (1-p)^{v-1}, \quad (17)$$

the discrete two-point binomial,

$$P(p=p_i) = f_i, \quad i=1,2, \quad (18)$$

and the normal-generated distribution

$$f(p) = \sigma^{-1} \exp\{\frac{1}{2}\mu^2 - \frac{1}{2}(\mu-p)^2/\sigma^2\} \quad (19)$$

where  $p = \Phi(-u)$ . In practice, it is important to know how accurately the prior distribution must be specified. Generally, the analyst does not possess sufficient information about the process to specify the prior with great confidence. Fortunately, most results indicate that precise specification of the prior is not critical, provided that a reasonable distribution is chosen. Continuous prior distributions are generally thought to be more appropriate than discrete ones, and the beta distribution (17) has been used extensively. However, when the underlying quality characteristic is a continuous variable that is normally distributed within each lot and the mean of this quality characteristic also has a normal prior, then the prior distribution for  $p$  has the form (19). The beta and normal-generated distributions can have very different shapes for the same mean and variance, and so significant differences in the optimal sampling plans may result. For further discussion of prior distributions, see Wetherill and Chiu [21].

#### The Major Approaches to Single Sampling for Attributes

Given a suitable prior distribution and a set of costs or losses associated with sampling plan operation, it is desirable to choose the sampling plan parameters that minimize the total cost. Perhaps the most widely used and detailed model is that of Guthrie and Johns [9]. A simplified form of this model is also presented by Hald [11]. The model is a linear cost model. All linear cost formulations lead to the same expected loss function. Hald utilizes the concept of break-even quality  $p_r$ , a fraction defective value at which it is just as costly to accept as to reject the lot. The expected loss per lot is

$$L = an + (N-n) \left\{ \int_0^{p_r} (p_r - p) [1 - P(p)] f(p) dp + \int_{p_r}^1 (p - p_r) P(p) f(p) dp \right\} \quad (20)$$

where  $a$  is a constant proportional to the variable cost of sampling and

$$P(p) = \sum_{d=0}^c \binom{n}{d} p^d (1-p)^{n-d}$$

is the probability of accepting a lot of quality  $p$ . Minimizing  $L$  with respect to  $n$  and  $c$  will produce the optimal sampling plan. This is sometimes called the Bayesian approach to designing a sampling plan.

The major focus of the research in this area has started with the Guthrie and Johns model. Hald has been a major contributor in the field, along with some of his coworkers. The major emphasis has been on finding asymptotic relationships between  $n$  and  $c$  for various process curves, and in producing tables suitable for use by professional practitioners. Hald [10] gives a number of significant findings in his 1960 paper. One part of the paper investigates the compound hypergeometric distribution; that is, the probability distribution of the number of defectives  $d$  in a random sample of size  $n$  given a prior distribution. The second part of the paper is directed towards actually finding optimum sampling plans for rectangular, beta, and double binomial priors. A general solution is given assuming the linear cost model and inequalities are given for  $n$  and  $c$ . In 1965, Hald [11] provided tables for the double binomial prior and in 1968, Hald [12] provided tables for the beta prior, along with asymptotic relationships between  $n$  and  $c$ , and between  $N$  and  $n$ .

Despite the significance of Hald's work, the tables he has provided are often difficult to use because of the large amount of information required. An alternative approach consists of formulating an appropriate cost model and optimizing it for a specific problem using direct search methods. Considerable work in this area has been done by G.K. Bennett, K.E. Case, and J.W. Schmidt and their students. For example, their 1972 and 1975 papers [3][17] develop economic models for single sampling plans for dealing simultaneously with multiple attributes. The cost models consist of a component representing inspection costs, a component representing the expected cost of lot rejection, and a component representing the expected cost of lot acceptance. Pattern search is used for model optimization. In general, direct search methods are a very effective approach for determining economically optimal acceptance sampling plans.

The economic impact of the disposition policy for rejected lots has also been investigated. While there are a number of possible lot disposition policies, the two cases that have been investigated most extensively are where rejected lots are either scrapped or screened (100 percent inspection). For work on lot disposition policies, see [18] and [20].

#### Other Work

There have been many other studies devoted to the economic design of acceptance sampling plans; for instance, a 1975 survey paper by Wetherill and Chiu [21] cites 253 references. While most of the work focuses on single sampling for attributes, there

has been some research on the economic design of more sophisticated sampling plans. For example, Stewart, Montgomery and Heikes [20] describe a procedure for the selection of double sampling plans for attributes based on prior distributions and costs. Models are presented for the cases where rejected lots are either screened or scrapped. They note that there is often little difference in cost between economically optimal single and double-sampling plans. However, when sampling costs are large, double sampling plans have much to offer. They also observe that arbitrary double-sampling plans, such as those in MIL STD 105D, may be very far from economically optimum.

The effect of inspection error on sampling plan design has also received considerable attention. If an inspector misclassifies good and bad items with constant probabilities, the effect is to translate the OC curve of the sampling plan, so that the actual or effective OC curve is somewhat different from the nominal or advertised OC curve. If the probabilities of misclassification are known, then one may directly incorporate this information into the economic design of the sampling plan. Generally, the presence of inspection errors implies that larger samples are necessary. A good review of the literature in this area is in Dorris and Foote [6].

Very little attention has been given to acceptance sampling by variables. Variables sampling is not as widely used in practice as attributes sampling for several reasons:

1. Variables measurement is often more expensive (difficult) than attributes measurement.
2. A separate plan must be used for each quality characteristic.
3. The estimation of fraction defective assumes a normally distributed quality characteristic. If this assumption is violated, the tail areas may be dramatically affected and the resulting fraction defective estimate grossly in error.
4. It is possible to reject a lot without actually finding any defectives, and this often upsets both producers and consumers.

However, remember that variables sampling can greatly reduce the required sample size, and that it does generally provide better information about the lot or process quality. For work on the economic design of variables sampling plan, see Ailor, Schmidt and Bennett [1], Schmidt, Bennett and Case [18], and Schmidt, Case and Bennett [19]. Reference [1] deals with the situation where the quality characteristics are a mixture of attributes and variables. In all of these studies, the approach taken is to formulate a cost model and optimize it via direct search methods.

#### SOME COMMENTS ON IMPLEMENTATION

The last 20 years have seen the development of numerous techniques for the design of process control and acceptance sampling schemes based on economic considerations. However, the indication is that very

very few practitioners have implemented any of these techniques (for example, see the surveys by Saniga and Shirland [16] and Chiu and Wetherill [5]). This is surprising, as most quality assurance managers claim that cost reduction and increased productivity is a major objective of their function. In many cases, an experienced engineer could design an appropriate technique, perhaps even one that is nearly economically optimal, but the use of a formal economic model to assist the analyst is a much more precise approach, leaving less to judgment. Furthermore, there is often a significant economic penalty associated with the "standard", judgment designs so frequently used in practice.

The implementation of these techniques requires a computer program of the cost model and the optimization procedure. The lack of availability of suitable computer software has certainly slowed practical implementation. This is an important gap between theory and practice that must be filled before the economic design of quality assurance techniques will become widespread.

Many practitioners are reluctant to use these techniques because of the difficulty in estimating costs, prior distributions, and other model parameters. Fortunately, costs and prior distributions do not have to be estimated with high precision, although some other parameters, such as the magnitude of the process shift, require more careful determination. Sensitivity analysis of the specific model could help the analyst discover which parameters are critical in his specific application. The availability of efficient, interactive computer software would be of significant value in this respect.

We will illustrate the use of a simple, interactive computer program for the optimal economic design of an  $\bar{x}$ -chart. The program assumes that the Duncan single-cause process model is appropriate, and requests the user to input values of  $a_1, a_2, a_3, a_4, \lambda, \delta, g$ , and  $D$ . The optimal control limit width  $k$ , interval between samples, and minimum cost per unit time, are calculated and displayed for a range of sample sizes, along with the  $\alpha$ -risk and power of the control chart for each  $(n, k, h)$  combination. The economically optimal control chart design may be found by inspection of the cost function values to find the minimum. The output provided enables the analyst to determine the sensitivity of the cost surface in the vicinity of the optimum. The program is written in FORTRAN for a CDC CYBER-74 computer, and involves less than 100 lines of code. Execution times are typically one second or less. Further details of the program are available from the author.

Consider a manufacturer of non-returnable glass bottles for packaging a carbonated soft drink beverage. The wall thickness of the bottles is an important quality characteristic. If the wall is too thin, internal pressure generated during filling will cause the bottle to burst. The manufacturer has been using  $\bar{x}$  and R charts with  $n=5, k=3$ , and taking samples every  $h=2$  hours to control the process. He wishes to compare this with an economically optimal design and estimate the savings.

Based on an analysis of the quality control

technicians' salaries, the costs of the test equipment, and the length of time required, it is estimated that the fixed cost of taking a sample is \$1.00, and the variable cost of sampling is approximately \$0.10 per bottle. It takes about one minute (0.0167 hours) to measure and record the wall thickness of a bottle.

The process is subject to several types of assignable causes resulting in large shifts, typically of about two standard deviations. (This is why the "standard"  $\bar{x}$ -chart design  $n=5$ ,  $k=3$  has been used - it is known to be reasonably effective for large shifts). The previous six months of line performance data indicate that the mean time between process breakdowns is about 20 hours. Thus  $\lambda=0.05$  is the parameter of the exponential distribution. The average time to investigate an action signal is one hour. Real alarms incur a \$25.00 search cost, on the average, while false alarms, which are more difficult to check out, incur a search cost of \$50.00.

The soft drink bottler to whom the bottles are sold has a policy of backcharging the bottle manufacturer for the costs of cleanup and lost production when an excessive number of defective bottles burst during filling. The past six months' experience indicates that \$100 per hour is a reasonable estimate of the penalty cost of operating in the out-of-control state.

Figure 1 shows the actual computer input and output for this example. Note that the optimum design has  $n=5$ ,  $k=2.99$ ,  $h=0.76$  hours (about 45 minutes), and a minimum cost of \$10.38 per hour. Thus the implication is that the manufacturer's existing control chart design is good with respect to  $n$  and  $k$ , but that he needs to sample more frequently. The actual cost of his current procedure is \$12.05 per hour, or about a 16 percent penalty cost. While the savings per hour seem small, this is a continuous production process operating three shifts per day, and the annual savings are over \$14,000.00.

After looking at the optimal  $\bar{x}$ -chart design, the bottle manufacturer suspects that he may have incorrectly estimated the penalty cost of out-of-control production ( $a_4$ ), and, at worst, he may have underestimated this parameter by 50 percent. Therefore, he reruns the program with  $a_4=\$150.00$  to investigate the effect of misspecifying this parameter. The computer output, shown in Figure 2, indicates that the optimum design is now  $n=5$ ,  $k=2.99$ ,  $h=0.62$ , with a minimum effect of increasing  $a_4$  by 50 percent is to decrease the interval between samples from 45 minutes to 37 minutes. The program could be used in this manner to quickly and easily investigate the effects of errors in specifying any of the model parameters. Based on this analysis, the manufacturer elects to adopt a 45 minute sampling interval, because it is administratively simple and it will not require any additional quality control technicians.

ENTER SYSTEM PARAMETERS:A1,A2,A3,A3PRIME,A4,LAMBDA,DELTA,G,D  
?

1.0,0.1,25.0,50.0,100.0,0.05,2.0,0.0167,1.0

0	N	OPTIMUM K	OPTIMUM H	ALPHA	POWER	COST
	1	2.30	.45	.0214	.3821	14.71
	2	2.52	.57	.0117	.6211	11.91
	3	2.68	.66	.0074	.7835	10.90
	4	2.84	.71	.0045	.8770	10.51
	5	2.99	.76	.0028	.9308	10.38
	6	3.13	.79	.0017	.9616	10.39
	7	3.27	.82	.0011	.9784	10.48
	8	3.40	.85	.0007	.9880	10.60
	9	3.53	.87	.0004	.9932	10.75
	10	3.66	.89	.0003	.9961	10.90
	11	3.78	.92	.0002	.9978	11.06
	12	3.90	.94	.0001	.9989	11.23
	13	4.02	.96	.0001	.9993	11.39
	14	4.14	.98	.0000	.9996	11.56
	15	4.25	1.00	.0000	.9998	11.72

.945 CP SECONDS EXECUTION TIME

Figure 1. Sample Computer Output

ENTER SYSTEM PARAMETERS: A1, A2, A3, A3PRIME, A4, LAMBDA, DELTA, G, D  
?

1.0, 0.1, 25.0, 50.0, 150.0, 0.05, 2.0, 0.0167, 1.0

O	N	OPTIMUM K	OPTIMUM H	ALPHA	POWER	COST
	1	2.31	.37	.0209	.3783	19.17
	2	2.52	.46	.0117	.6211	15.71
	3	2.68	.54	.0074	.7835	14.48
	4	2.84	.58	.0045	.8770	14.01
	5	2.99	.62	.0028	.9308	13.88
	6	3.13	.65	.0017	.9616	13.91
	7	3.27	.67	.0011	.9784	14.04
	8	3.40	.69	.0007	.9880	14.21
	9	3.53	.71	.0004	.9932	14.41
	10	3.66	.73	.0003	.9961	14.62
	11	3.78	.75	.0002	.9978	14.84
	12	3.90	.77	.0001	.9988	15.06
	13	4.02	.78	.0001	.9993	15.28
	14	4.14	.80	.0000	.9996	15.50

.417 CP SECONDS EXECUTION TIME

Figure 2. Sample Computer Output

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